Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

0123456789

FURTHER MATHEMATICS **9231/04**

Paper 4 Further Probability & Statistics **For examination from 2020**

SPECIMEN PAPER **1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

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1 **(a)** State briefly the circumstances under which a non-parametric test of significance should be used rather than a parametric test. [1]

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The level of pollution in a river was measured at 12 randomly chosen locations. The results, in suitable units, are shown below, where higher values represent greater pollution.

5.62 5.73 6.55 6.81 6.10 5.75 5.87 6.47 5.86 6.26 6.99 5.91

(b) Use a Wilcoxon signed-rank test to test whether the average pollution level in the river is more than 6.00. Use a 5% significance level. [6]

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2 Each of 200 identically biased dice is thrown repeatedly until an even number is obtained. The number of throws needed is recorded and the results are summarised in the following table.

Number of throws	1	2	3	4	5	6	$\geqslant 7$
Frequency	126	43	22	3	5	1	0

Carry out a goodness of fit test, at the 5% significance level, to test whether Geo(0.6) is a satisfactory model for the data. [7]

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3 Employees at a particular company have been working seven hours each day, from 9 am to 4 pm. To try to reduce absence, the company decides to introduce 'flexi-time' and allow employees to work their seven hours each day at any time between 7 am and 9 pm. For a random sample of 10 employees, the numbers of hours of absence in the year before and the year after the introduction of flexi-time are given in the following table.

Employee	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Before	42	35	96	74	20	5	78	45	146	0
After	34	32	100	72	31	2	61	35	140	0

Test, at the 10% significance level, whether the population mean number of hours of absence has decreased following the introduction of flexi-time, stating any assumption that you make. [8]

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4 The number, x, of a certain type of sea shell was counted at 60 randomly chosen sites, each one metre square, along the coastline in country A. The number, y, of the same type of sea shell was counted at 50 randomly chosen sites, each one metre square, along the coastline in country B. The results are summarised as follows, where $\overline{x}$ and $\overline{y}$ denote the sample means of x and y respectively.

$$\overline{x} = 29.2 \quad \Sigma(x - \overline{x})^2 = 4341.6 \quad \overline{y} = 24.4 \quad \Sigma(y - \overline{y})^2 = 3732.0$$

Find a 95% confidence interval for the difference between the mean number of sea shells, per square metre, on the coastlines in country A and in country B. [7]

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5 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} 0 & x < 0, \\ \frac{6}{5}x & 0 \leqslant x \leqslant 1, \\ \frac{6}{5}x^{-4} & x > 1. \end{cases}$$

(a) Find $\mathrm{P}(X > 1)$. [1]

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(b) Find the median value of X. [2]

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(c) Given that $E(X) = 1$, find the variance of X. [3]

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(d) Find $E\left(\sqrt{X}\right)$. [2]

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6 Aisha has a bag containing 3 red balls and 3 white balls. She selects a ball at random, notes its colour and returns it to the bag; the same process is repeated twice more. The number of red balls selected by Aisha is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X. [2]

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Basant also has a bag containing 3 red balls and 3 white balls. He selects three balls at random, without replacement, from his bag. The number of red balls selected by Basant is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y. [3]

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The random variable Z is the total number of red balls selected by Aisha and Basant.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

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(d) Use the probability generating function of Z to find $\mathrm{E}(Z)$ and $\mathrm{Var}(Z)$. [5]

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Additional page

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